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REVIEWS

Mathematical Biofluiddynamics. By M. J. LIGHTHILL. Society for Industrial and Applied Mathematics, 1975. 281 pp. $21.50.

"...About 10^9 years of animal evolution in an aqueous environment, by preferential retention of specific variations that increase ability to survive and produce fertile offspring, have inevitably produced rather refined means of generating fast movement at low energy cost, which merit study." With characteristic verve and stylistic elegance M. J. Lighthill thus pinpoints the importance of aquatic animal propulsion, a topic constituting about half of his monograph. This is a written version of the author's lectures at a "Mathematical Biofluiddynamics" research conference at Rensselaer Polytechnic Institute in 1973. Part of the material is reprinted from earlier or contemporary articles, but much is new.

The monograph is organized into two main portions, external and internal biofluiddynamics. In the former category, considerable material on animal flight supplements the treatment of aquatic animal propulsion. Substantial discussions of respiratory flow and pulse propagation make up the major part of the second category. In addition there are briefer qualitative discussions without equations on the connexion between blood flow and arterial disease and on microcirculation.

The most highly developed portion of the theory concerns fish swimming. The goal of obtaining deep understanding of how a finny reptile propels itself through wet water precludes tidy formalism, so physical reasoning is repeatedly used to extract illuminating approximations. Students of applied mathematics will find especially valuable the chronicle of how earlier, relatively crude theories have been succeeded by more accurate ones, with the 'last word' still a matter for the future.

The first major advance in the study of fish propulsion was made by G. J. Hancock (a former student of Lighthill) and the zoologist James Gray. Their *resistive* theory regarded "the interaction between a short segment of the elongated animal and the surrounding aqueous medium as consisting of a localised resistive force depending only upon the relative velocity between them, but with components of that relative velocity tangential to the line of centres resisted less then components normal to the line of centres". Later Lighthill developed a more accurate *reactive* approach to the motion of elongated fish. This is a slender-body theory, but with a special zoological feature that "any unbalanced side forces in the posterior region of the fish may produce wasteful, undesired 'recoil' motions of sideslip of the fish's mass centre, and yaw about it". Extension by Lighthill of this theory to large amplitude motions was taken up by his student D. Weihs, especially in connexion with starting and turning manoeuvres.

A place of honour in biofluiddynamics is already assured for the parasite wasp *Encarsia formosa* and its mentor T. Weis-Fogh. The latter showed that the former hovers in a superior fashion by CLAPPING its wings together behind it, effectively forming a single body, then FLINGING the wings open to a V position before breaking them apart in a normal hovering movement. (The capital letters only faintly indicate the enormous gusto with which the author demonstrated the movements in his lectures!) Weis-Fogh showed that at work here is a fundamentally new method of lift generation – new to scientists, not to wasps – which is based on an inviscid two-dimensional flow. "To be sure", Lighthill comments, "for inviscid two-dimensional flow the doctrines of Helmholtz, Stokes and Kelvin tell us that a body starting to move in fluid at rest retains always the same zero circulation of fluid around it, preventing the generation of lift on the body... Those doctrines do not, however, rule out the possibility that when the body *breaks into two pieces* there may be equal and opposite circulations round them, each suitable for generating the lift required in the pieces' subsequent motions!"

Chapter 9 (reprinted from this *Journal*) presents the Weis-Fogh theory as extended by Lighthill. The theory starts with a classical conformal mapping approach to the two-dimensional inviscid problem, adds considerations of viscosity and three-dimensionality, and concludes that the far field would be that of the Landau–Squire laminar round jet due to a point source of momentum in a viscous fluid. The novelty of the phenomenon and an attendant theory finely balanced between comprehensiveness and simplicity make this chapter a special delight.

Some other highlights will only be mentioned:

(i) the substantial ratio between oscillatory motion and steady-flow resistive drags, permitting theoretical demonstration (by Weihs) that to minimize energy consumption neutrally buoyant fish should alternate active swimming with rigid gliding, while heavier-than-water fish should alternate active climbing with downward gliding (p. 147),

(ii) five unusual characteristics of flows in the human body (pp. 200 ff.),

(iii) the highly non-Poiseuille character of flow in human bronchi (p. 217),

(iv) the largely unexplained valveless one-way flow through the gas exchange units of birds (pp. 217 ff.).

Two deficiencies must also be cited.

(i) Unfortunately, the perennial British custom from pure mathematics remains hardy; there is no index.

(ii) Although the author's many indications of open problems will generally be received with enthusiasm by researchers, the reviewer was disappointed by one omission. Disparagement of "the feeble swimming attempts of *Homo sapiens*" hardly justifies failure to carry out research on human swimming by one whose natatory prowess leads him to such feats as the circumnavigation of good sized islands in the English channel!

As concluding remarks we note that this volume is not at all a treatise, but a lively account of a developing subject, enlivened by a few important and candid personal remarks. Thus the author reports that "work of ours at Manchester on motions at very low Reynolds numbers failed to be influential because we had not learned the art of communicating with zoologists: either to discover from them the problems of zoological significance or to indicate to them our mechanically significant conclusions". He stresses that effective biofluid-dynamics research demands closer collaboration between theorists and observer-experimenters than might be usual in more classical fields of fluid mechanics. His monograph displays numerous successful fruits of such collaboration and indicates that many others are there for the picking.

L. A. SEGEL

Cours de Mécanique des Milieux Continus. Tome 1. Théorie générale. By P. GERMAIN. Masson et Cie, 1973. 417 pp.

This is the first volume of a projected series by the author, at the current undergraduate level at the University of Paris. It also seems well suited to Honours courses in applied mathematics in British universities.

The author states that he is not aiming at a systematic and logically ordered treatise, but rather a progession in keeping with pedagogic needs. Thus, while this introductory volume is nominally about fundamentals common to all theories of materials, it does not pursue generalities much beyond the level of an undergraduate second year; moreover, it includes a fair number of elementary illustrative problems drawn from viscometry and metallurgical testing. There is also a substantial appendix on vectors and tensors, and an extensive collection of exercises. Subsequent volumes will cover various branches in greater depth, beginning with elasticity, and proceeding to fluid mechanics, viscoelasticity, plasticity, etc.

The author's approach is in the spirit of present-day continuum mechanics, and includes for instance the theory of finite strain (as far as polar decomposition); the Piola–Lagrange stress tensor; various fluxes of stress; and objective formulations of constitutive relations. Among the more distinctive features are (i) an unusually diverse collection of conservation formulae for convected volumes and surfaces, including discontinuities, (ii) a comprehensive treatment of thermal effects, both static and dynamic, (iii) a unified presentation of constitutive laws from the standpoint of convex potentials and dissipative functions, coupled with the axiom of orthogonality (still controversial, especially in the plastic and viscous contexts), and (iv) an extended discussion of the basic properties of convex functions.

The exposition throughout is scholarly and precise, with careful attention to the finer points of analysis. This is an excellent introduction to continuum mechanics, and it will be interesting to see how the Course develops in succeeding volumes.

R. HILL